AC Voltage Measurements: Why does a 1V square wave measure 1.111V?

Rick Nungester WA6NDR, 4/9/20

- Introduction
  - Handheld Multimeters: Harbor Freight ($6) to Keysight ($480, "True RMS").
  - Computer Algebra Systems (CASs) and Derive.
  - Digilent Electronics Explorer Board and Waveforms Software.
- Average Voltage of a Full Wave Rectified Sine Wave
  - Derive file, math, graphical meaning of the calculus: Rectify and Average.
  - "Average" in hardware is a low-pass filter. In math, spread out the area under the humps (calculus) evenly. The calculus expression can be read as "find the area under the full wave rectified humps and spread it out flat."
  - Results: For this 1 Vpk sine wave the area spreads out to a height of 2/pi ~= 0.6366V. **For a full wave rectified sine wave Vavg = (2/pi) * Vpk**.
  - What about half wave rectifying? It is no problem given a symmetric waveform (the same below and above the zero axis). For any half wave rectified sine wave Vavg = 1/2 the full wave case = (1/pi) * Vpk ~= 0.3183 * Vpk.
  - **Demo:** Sine, half wave rectified, full wave rectified, both filtered.
- RMS Voltage of a Sine Wave
  - RMS = Root Mean Squared = the square **Root** of the **Mean** of the voltage **Squared**. Or in the order of what must be done, **square** the waveform, then find the **mean** (or average or filtered) value, then take the square **root**. Why?! Ohm's Law for power: P = V^2/R. Power is proportional to voltage squared. Average the voltage squared to get average power. Take the square root to get **the DC voltage that would produce the same power as the original waveform**.
  - RMS voltage is hard to measure. Analog methods require either squaring and square root circuits, or heat measurements using a thermocouple. Digital methods require sampling and math to square, filter, and square root.
  - Derive file, math, and graphs. Square, mean, root. Trigonometry: A sine wave squared is a sine wave at twice the frequency, shifted up by Vpk/2. Average. (Same as "mean" or "filter"). Take the area under the humps and spread it out. Square root.
  - Results: For this 1 Vpk sine wave the area spreads out to a height sqrt(2) / 2 ~= 0.7071V. **In general for a sine wave Vrms = (sqrt(2) / 2) * Vpk**.
  - **Demo:** Sine, add traces for Squared, Filtered, Square Root.
- Average Detected RMS Calibrated Voltage (Vavg_rms) of a Sine Wave
  - Vavg is easy to get, but we want Vrms. **Assuming a sine wave** how do we get Vrms from Vavg?

- Derive file, math, Vavg_rms = (sqrt(2) * pi / 4) * Vavg ~= 1.111 * Vavg.
- **This is what your AC Voltmeter likely probably does. It detects the full wave rectified average voltage and multiplies it by 1.111**. Or it detects the half wave rectified average voltage and multiplies it by 2.222.

- How about triangle and square waves? (See the Derive math file.)
  - For average remember "full wave rectify, filter".
  - For RMS remember "square, filter, square root".
  - **Demo:** Triangle and Square, average and RMS voltages.
- How about a non-symmetric complex waveform
  - 0.5 Vpk 1 kHz + 0.5 Vpk 2 kHz with a 90-degree phase shift (Derive file math).
  - **Demo:** Switch to this complex waveform. (Peaks add to 1V, subtract to 0V.)
  - Nine voltage measurements all with different values.
    - Peak+ 1.000
    - Peak- 0.562
    - Peak-to-Peak 1.562
    - (Peak+ + Peak-)/2 0.781
    - Average 0.000
    - Average Rectified 0.413
    - RMS 0.500
    - RMS * sqrt(2) 0.707
    - Avg Det RMS Cal 0.459
- Summary of results for 1 Vpk waveforms

	Vavg	Vrms	Vavg_rms
sine	0.637	**0.707**	**0.707**
triangle	0.500	0.577	0.555
square	1.000	1.000	1.111
complex	0.413	0.500	0.459

- Using an average detecting RMS calibrated meter, to measure the RMS value of a triangle wave multiply the meter reading by 0.577/0.555 = 1.04. To measure the RMS value of a square wave multiply the meter reading by 1.000/1.111 = 0.900.

#1: Average, RMS & Average Detected RMS Calibrated Voltage

#2: Rick Nungester, 6/8/16 (updated 4/5/20)

#3: Examples are all 1 Volt peak with period 2*pi.

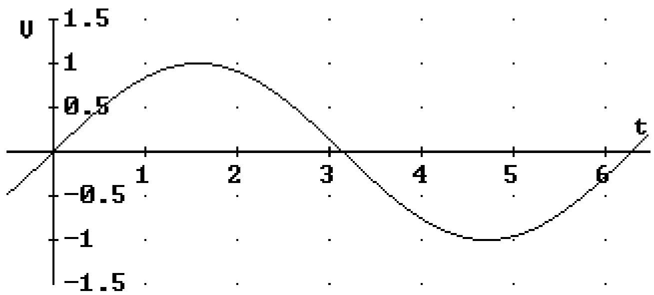
#4: CaseMode := Sensitive

#5: InputMode := Word

#6: **

#7: Average value of a full wave rectified sine wave

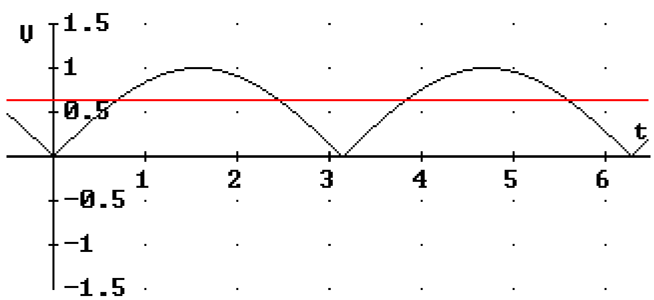
#8: s(t) := SIN(t)



#9: $$\frac{1}{2\cdot\pi}\cdot\int_{0}^{2\cdot\pi} |s(t)|\ dt$$

#10: $$\frac{2}{\pi}$$

#11: 0.6366197723



#12: **

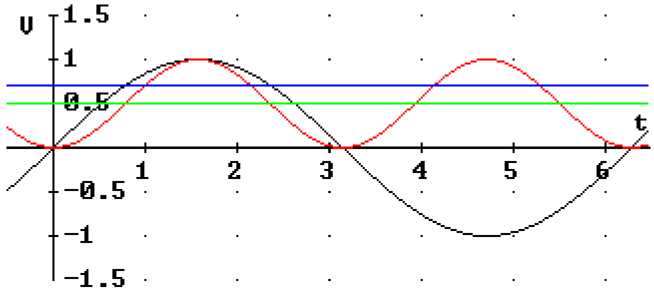
#13: RMS value of a sine wave

#14: RMS = Root Mean Squared (square, average, square root)

#15: $$\sqrt{\left(\frac{1}{2\cdot\pi}\cdot\int_{0}^{2\cdot\pi} s(t)^{2}\ dt\right)}$$

#16: $$\frac{\sqrt{2}}{2}$$

#17: 0.7071067811



#18: ***

#19: Average Detected RMS Calibrated value of a sine wave

#20: FOR A SINE WAVE, multiply the average by what to get RMS?

#21: $$\text{SOLVE}\left(\frac{2}{\pi}\cdot x = \frac{\sqrt{2}}{2},\ x\right)$$

#22: $$x = \frac{\sqrt{2}\cdot\pi}{4}$$

#23: $$x = 1.110720734$$

#24: Average Detected RMS Calibrated = rectify, average, *sqrt(2)*pi/4

#25: $$\text{Vavg\_rms(Vavg)} := \text{Vavg}\cdot\frac{\sqrt{2}\cdot\pi}{4}$$

#26: Check: Avg Det RMS Cal of a sine wave should be sqrt(2)/2 ~= 0.707.

#27: $$\text{Vavg\_rms}\left(\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} |s(t)|\ dt\right)$$

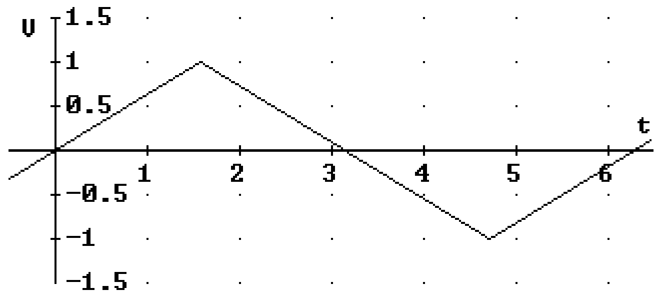
#28: $$\frac{\sqrt{2}}{2}$$

#29: 0.7071067811

#30: ***

#31: Average value of a full wave rectified triangle wave

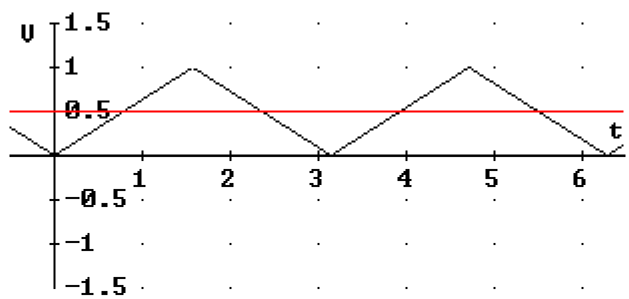
#32: $$\mathrm{tri}(t) := \frac{\left|4\cdot\pi\cdot \mathrm{FLOOR}\left(\frac{t}{2\cdot\pi} + \frac{3}{4}\right) - 2\cdot t - \pi\right|}{\pi} - 1$$



#33: $$\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} |\mathrm{tri}(t)|\ dt$$

#34: $$\frac{1}{2}$$

#35: 0.5



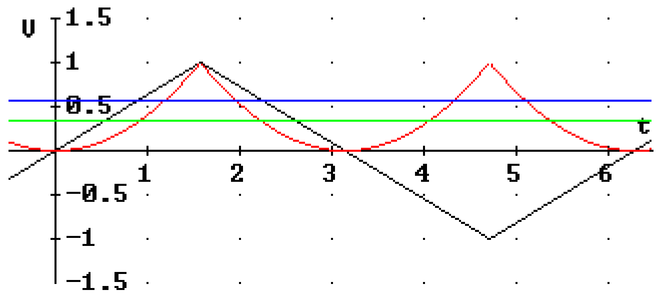
#36: ***

#37: RMS value of a triangle wave

#38: $$\sqrt{\left(\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} \mathrm{tri}(t)^2\ dt\right)}$$

#39: $$\frac{\sqrt{3}}{3}$$

#40: 0.5773502691



#41: **

#42: Average Detected RMS Calibrated value of a triangle wave

#43: $\text{Vrms}\left(\frac{1}{2\cdot\pi}\cdot\int_{0}^{2\cdot\pi} |\text{tri}(t)|\, dt\right)$

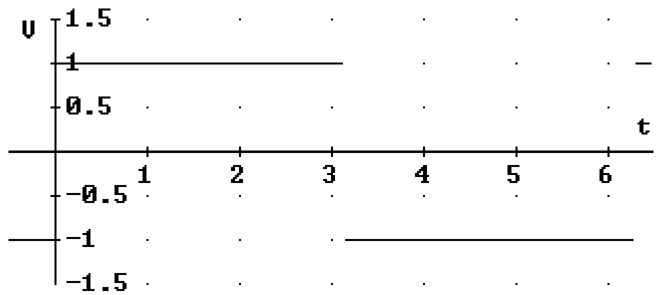
#44: $$\frac{\sqrt{2}\cdot\pi}{8}$$

#45: $$0.5553603672$$

#46: **

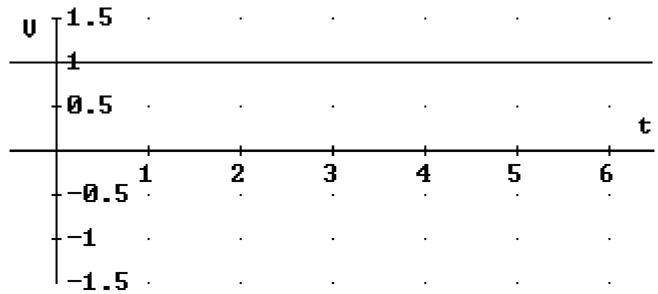
#47: Average value of a full wave rectified square wave

#48: square(t) := SIGN(SIN(t))



#49: $\frac{1}{2\cdot\pi}\cdot\int_{0}^{2\cdot\pi} |\text{square}(t)|\, dt$

#50: $$1$$

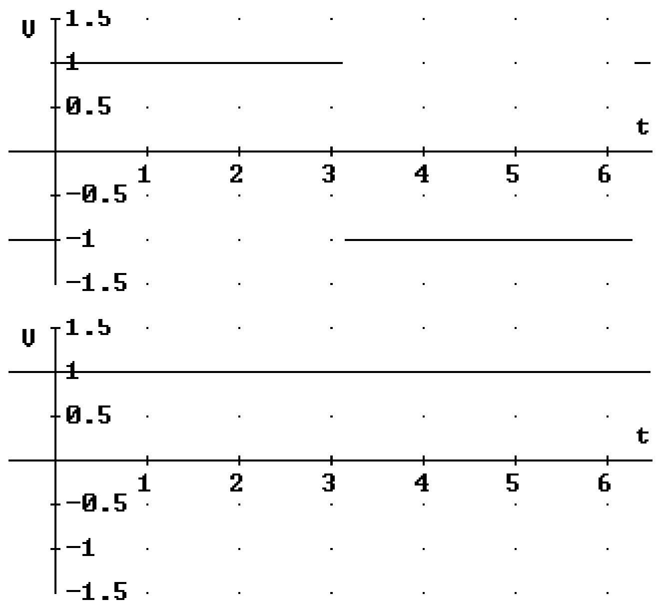


#51: **

#52: RMS value of a square wave

#53: $$\sqrt{\left(\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} \mathrm{square}(t)^2\, dt\right)}$$

#54: $$1$$



#55: **

#56: Average Detected RMS Calibrated value of a square wave

#57: $$\mathrm{Vavg\_rms}\left(\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} |\mathrm{square}(t)|\, dt\right)$$

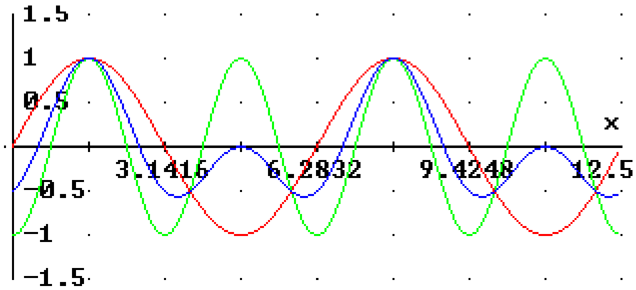
#58: $$\frac{\sqrt{2}\cdot\pi}{4}$$

#59: $$1.110720734$$

#60: **

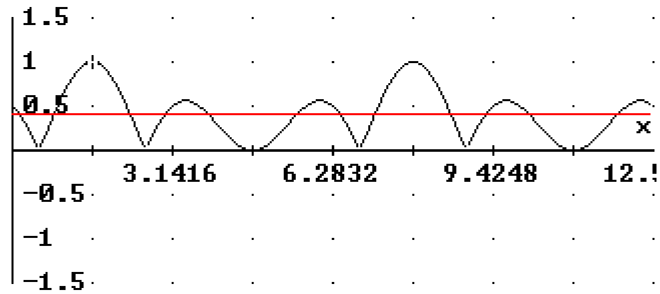
#61: Average value of a full wave rectified complex waveform

#62: $$c(t) := \frac{\mathrm{SIN}(t) + \mathrm{SIN}\left(2\cdot t - \frac{\pi}{2}\right)}{2}$$



#63: $\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} |c(t)|\ dt$

#64: 0.4134966721



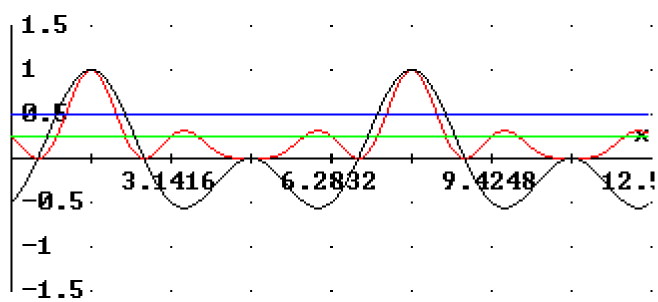
#65: ***

#66: RMS value of a complex waveform

#67: $\sqrt{\left(\frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} c(t)^2\ dt\right)}$

#68: $\frac{1}{2}$

#69: 0.5



#70: ***

#71: Average Detected RMS Calibrated value of a complex waveform

#72: $$\text{Vavg\_rms}\left(\frac{1}{2\cdot\pi}\cdot\int_{0}^{2\cdot\pi}\left|c(t)\right|\,dt\right)$$

#73: 0.4592793274